A Gradient Based Optimization Algorithm for Fuel Cell Stack Diagnostics from External Magnetic Field Measurements

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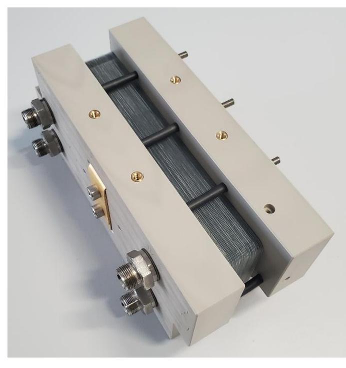
Abstract

This article presents a new inverse algorithm based on magnetic tomography for fuel cell stack diagnostics. The aim is to determine the local internal resistivities of fuel cell stacks from external magnetic field measurements. The inverse problem is solved by minimizing the difference between the simulated magnetic field and the measured magnetic field. Sensitivities are calculated using the adjoint state method, and numerical results are presented shown on a 3D-fault.

Scope

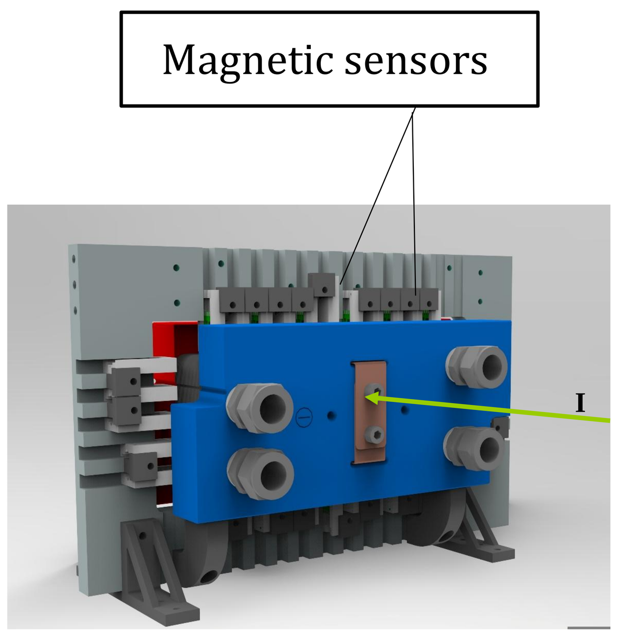
Hydrogen Fuel Cell Stacks
as part of the Energy Transition
Oxygen + Hydrogen ➡ Electricity + Heat + Water
Hurdles such as:
Reduced **reliability** and **performance** losses
Caused by: Drying / Flooding
Fuel / oxidant starvation
Catalyst degradation
Catalyst distribution



Diagnostic Tools are needed which are:
1. Non-invasive
2. Able to localize faults

} **Magnetic tomography**

Magnetic Tomography



Inverse Problem Formulation

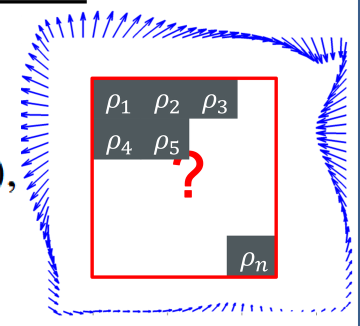
Find local fault by identifying local resistivities such that:
1. Measured Magnetic signature and of the Electrocinetic Model fit
2. Model voltage respects Measured Stack Voltage

Formulation of the inverse Problem as an optimization approach:

$$\widehat{\boldsymbol{\rho}} = \arg\min_{\boldsymbol{\rho}} \left(\left\| \boldsymbol{B_m} - \boldsymbol{B_s}(\boldsymbol{J}(\boldsymbol{\rho})) \right\|^2 + \alpha \left\| \mathbf{L}\boldsymbol{\rho} \right\|^2 \right)$$

$$\text{s.t.} \quad U_s(\boldsymbol{\rho}) - U_m = 0 \quad \text{(Nonlinear Constraint)},$$

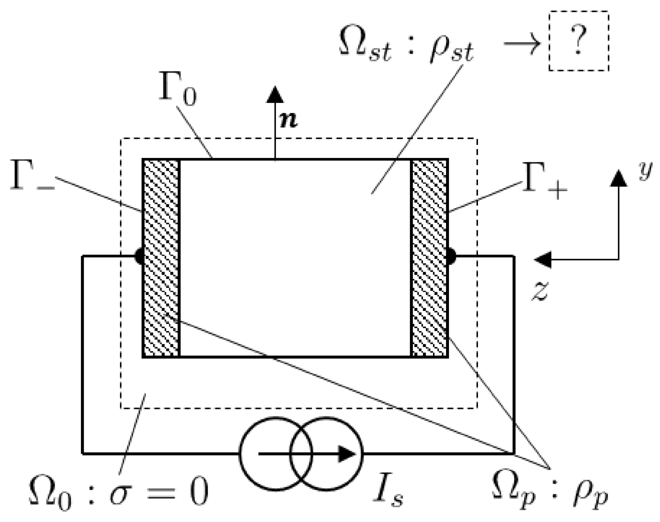
$$\boldsymbol{\rho_{min}} \leq \boldsymbol{\rho} \leq \boldsymbol{\rho_{max}} \quad \text{(Bounds)}.$$



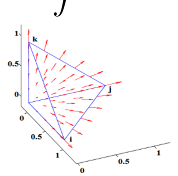
Challenge:
Compute derivative of cost function and constraints satisfying Electrokinetic equations with imposed current (FEM-Model)

Inverse Method by non-linear Optimization

1. FEM-Modelling at Constant J



- Electrokinetic equations: $\boldsymbol{E} = -\nabla U \quad \boldsymbol{E} = \rho \cdot \boldsymbol{J} \quad \nabla \cdot \boldsymbol{J} = 0$
- Boundary conditions: $\Gamma_- : \int J_n = -I_s \;\; \Gamma_+ : \int J_n = I_s \;\; \Gamma_0 : \int J_n = 0$
- *J* interpolation on face shape functions (Whitney 2-form) $\boldsymbol{J} = \sum_j \omega_j I_j$



- Galerkin projection on test shape function: $\int_\Omega \omega_i \cdot \rho \cdot \boldsymbol{J} \, \mathrm{d}\Omega = -\int_\Omega \omega_i \nabla U \, \mathrm{d}\Omega$
- Integration of Impedance matrix system

$$\mathbf{R}\boldsymbol{I} = \boldsymbol{U} \qquad \boldsymbol{U}_i = -\int_\Omega \boldsymbol{\omega}_i \nabla U \, \mathrm{d}\Omega \qquad \mathbf{R}_{i,j} = \int_\Omega \boldsymbol{\omega}_i \boldsymbol{\omega}_j \rho \, \mathrm{d}\Omega$$

- Free divergence of *J* obtained with incidence matrix $\mathbf{M_s}$ and $\mathbf{M_R}$:

$$\underbrace{\begin{bmatrix} \mathbf{M_R R}(\boldsymbol{\rho})\mathbf{M_R}^{\mathrm{T}} & \mathbf{M_S} \\ \mathbf{M_S}^{\mathrm{T}} & \mathbf{0} \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} I_{\text{loop}} \\ U_{\mathrm{S}} \end{bmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{bmatrix} 0 \\ I_{\mathrm{S}} \end{bmatrix}}_{\boldsymbol{S}_{\mathrm{H}}}$$

- Computation of $\boldsymbol{B}$ with integration of Biot-Savart law $\boldsymbol{B} = \frac{\mu_0}{4\pi} \int_\Omega \boldsymbol{J}(\boldsymbol{\rho}) \times \frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} \mathrm{d}\Omega \Rightarrow \boldsymbol{B}_{\mathrm{s}} = \mathbf{A}\, \boldsymbol{J}(\boldsymbol{\rho})$

2. Compute Sensitivities with **Adjoint Method**[2]

1. Objective Function:

$$r(\boldsymbol{J}(\boldsymbol{\rho})) = ||\boldsymbol{B_m} - \mathbf{A}\boldsymbol{J}(\boldsymbol{\rho})||^2$$

$$\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\rho}}(r(\boldsymbol{J}(\boldsymbol{\rho}))) = \underbrace{\frac{\partial r}{\partial \boldsymbol{\rho}}}_{=0} + \frac{\partial r}{\partial \boldsymbol{J}^{\mathrm{T}}} \frac{\partial \boldsymbol{J}}{\partial \boldsymbol{\rho}} \Rightarrow \frac{\mathrm{d}r(\boldsymbol{\rho})}{\mathrm{d}\boldsymbol{\rho}} = \frac{\partial r}{\partial \boldsymbol{x}^{\mathrm{T}}} \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{\rho}}$$

Step 1:

$$\underbrace{\begin{bmatrix} \mathbf{M_R R}(\boldsymbol{\rho})\mathbf{M_R}^{\mathrm{T}} & \mathbf{M_S} \\ \mathbf{M_S}^{\mathrm{T}} & \mathbf{0} \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} \frac{\mathrm{d}\boldsymbol{I}_{\text{loop}}}{\mathrm{d}\boldsymbol{\rho}} \\ \frac{\mathrm{d}U_S}{\mathrm{d}\boldsymbol{\rho}} \end{bmatrix}}_{\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}\boldsymbol{\rho}}} = -\underbrace{\begin{bmatrix} \mathbf{M_R} \frac{\mathrm{d}\mathbf{R}(\boldsymbol{\rho})}{\mathrm{d}\boldsymbol{\rho}} \mathbf{M_R}^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}_{-\frac{\mathrm{d}\mathbf{K}}{\mathrm{d}\boldsymbol{\rho}}} \underbrace{\begin{bmatrix} \boldsymbol{I}_{\text{loop}} \\ U_S \end{bmatrix}}_{\boldsymbol{x}}$$

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}\boldsymbol{\rho}} = \boxed{-\mathbf{K}^{-1}} \frac{\mathrm{d}\mathbf{K}}{\mathrm{d}\boldsymbol{\rho}} \boldsymbol{x}$$

Costly! → Use Adjoint State Method

Step 2: $\mathbf{K}^{\mathrm{T}}\boldsymbol{\lambda} = \frac{\mathrm{d}r}{\mathrm{d}\boldsymbol{x}} \rightarrow \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}\boldsymbol{\rho}} = -\mathbf{K}^{-1}\frac{\mathrm{d}\mathbf{K}}{\mathrm{d}\boldsymbol{\rho}}\boldsymbol{x}$ $\quad \boxed{\frac{\mathrm{d}r(\boldsymbol{\rho})}{\mathrm{d}\boldsymbol{\rho}} = -\boldsymbol{\lambda}\frac{\mathrm{d}\mathbf{K}}{\mathrm{d}\boldsymbol{\rho}}\boldsymbol{x}}$

Step 3: $\frac{\partial}{\partial \boldsymbol{J}}(r(\boldsymbol{J}(\boldsymbol{\rho}))) = 2(\mathbf{A}^{\mathrm{T}}\mathbf{A}\boldsymbol{J}(\boldsymbol{\rho}) - (\boldsymbol{B}_m^{\mathrm{T}}\mathbf{A})^{\mathrm{T}})$

2. Constraint Function analog to objective function:

$$0 = U_s(\boldsymbol{\rho}) - U_m$$

$$\frac{\mathrm{d}U_s(\boldsymbol{\rho})}{\mathrm{d}\boldsymbol{\rho}} = \frac{\partial U_s}{\partial \boldsymbol{x}^{\mathrm{T}}} \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{\rho}}$$

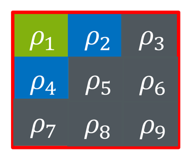
Set adjoint state:

$$\mathbf{K}^{\mathrm{T}}\boldsymbol{\lambda}_{U_s} = \frac{\partial U_s}{\partial \boldsymbol{x}} = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$$

$$\boxed{\frac{\mathrm{d}U_s(\boldsymbol{\rho})}{\mathrm{d}\boldsymbol{\rho}} = -\boldsymbol{\lambda}_{U_s}\frac{\mathrm{d}\mathbf{K}}{\mathrm{d}\boldsymbol{\rho}}\boldsymbol{x}}$$

3. Regularization

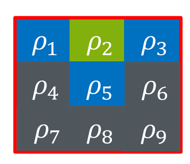
Problem is ill-posed
→ Supposition of regular ρ in the beginning of the FC-Stack life
→ Add regularization term
→ Chose α with L-Curve method[4]



$$\mathbf{L} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \dots \\ & & & & & \vdots \\ & & & & & \dots \end{bmatrix}$$

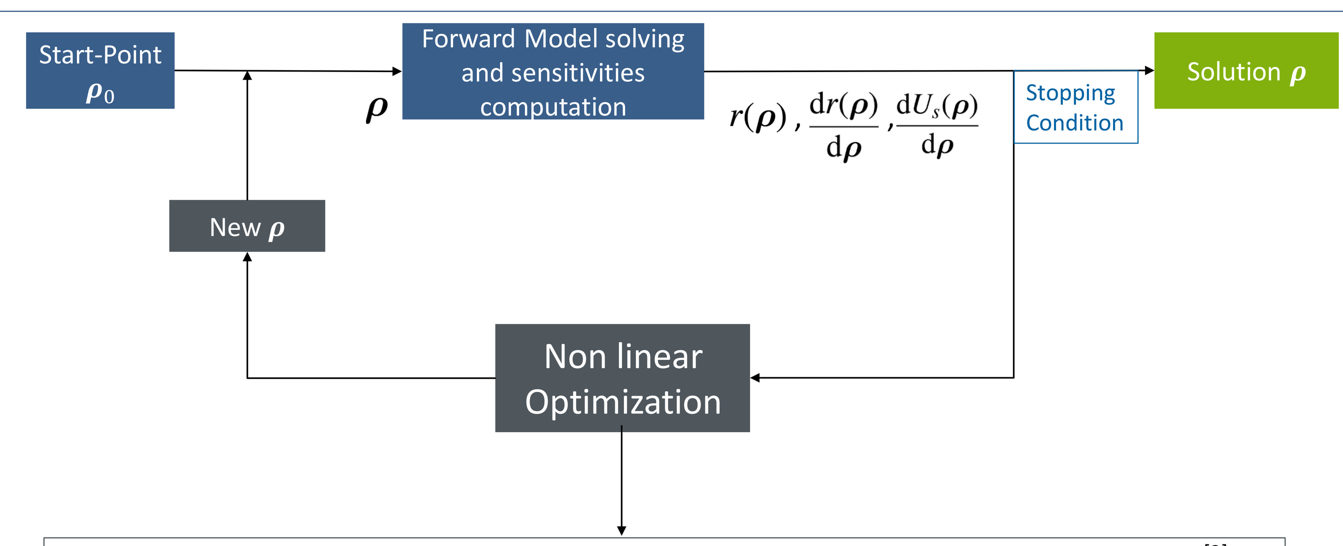
$$\boldsymbol{Y}(\boldsymbol{\rho}) = \alpha||\mathrm{L}\boldsymbol{\rho}||^2$$

$$\frac{\partial \boldsymbol{Y}(\boldsymbol{\rho})}{\partial \boldsymbol{\rho}} = 2\alpha \mathrm{L}^{\mathrm{T}}\mathrm{L}\boldsymbol{\rho}$$



$$\mathbf{L} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \dots \\ -\frac{1}{3} & 1 & -\frac{1}{3} & 0 & -\frac{1}{3} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

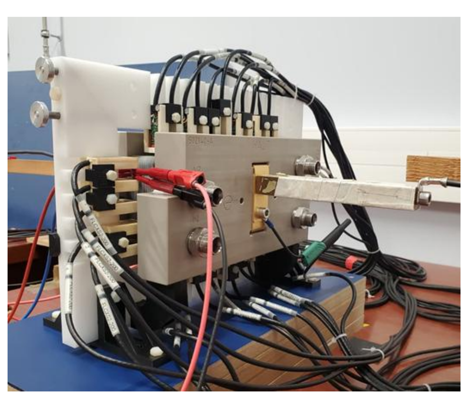
4. Optimization Algorithm



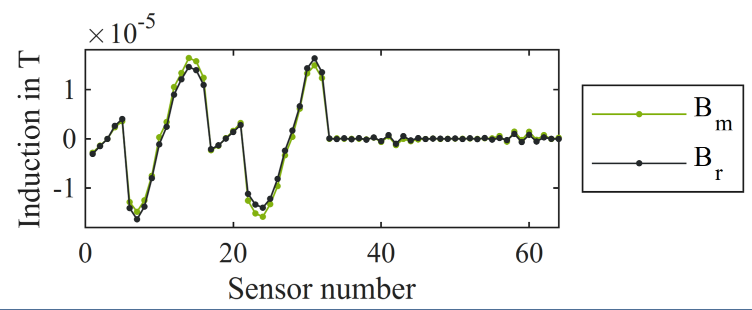
Interior Point Algorithm → Compose Lagrangian of cost function, constraints and bounds[3]
Gradient Descend → minimize Lagrangian[2]
Broyden-Fletcher-Goldfarb-Shanno (BFGS) → Hessian approximation[3]

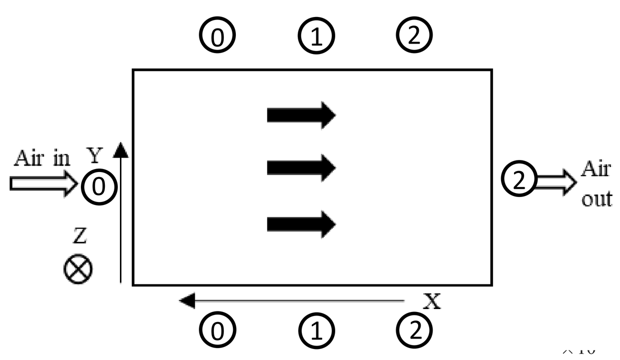
Results

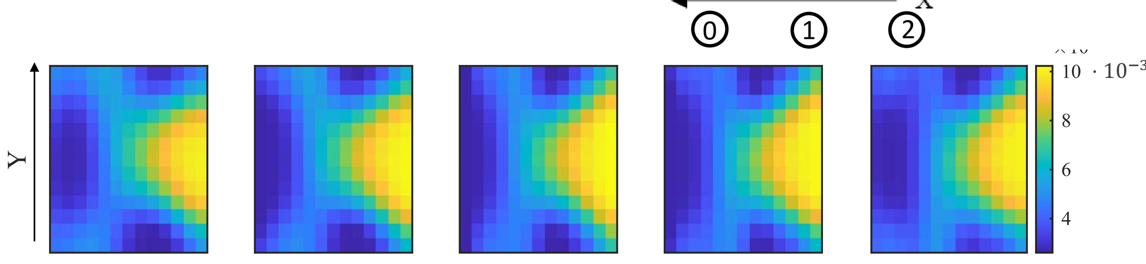
Real Measurement on a False Stack (only BPP and GDL)

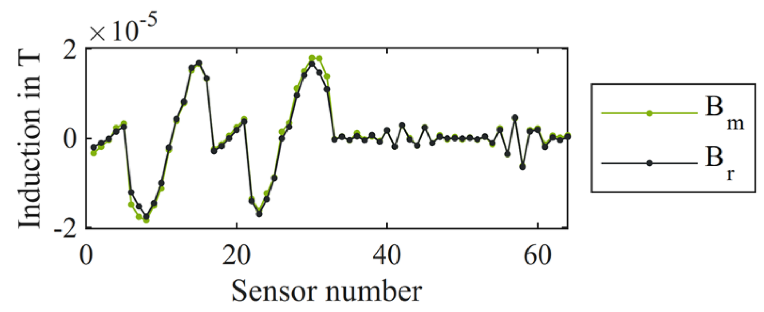


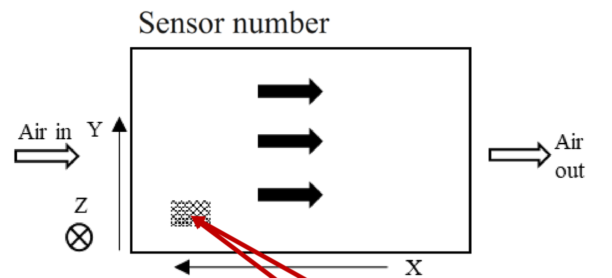
Procedure fault placement:
a) Unscrew screws by (0) 0°, (1) 45° and 90° (2)
b) Local fault placed on membranes 20-21 inside the stack
1. Magnetic measurement
2. Data Treatment
3. Apply inverse method on data

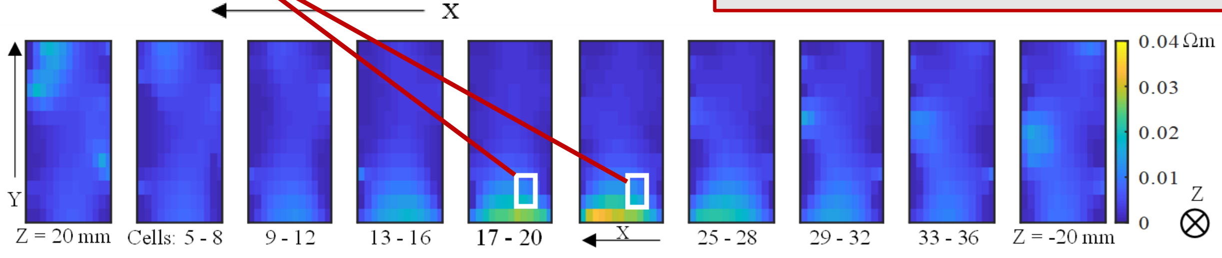












- Gradient successfully reconstructed a) ☺
- A fault is successfully found b) ☺
- Fault is supposed to be in the two white rectangles ☹
- Fault is not at the correct position ☹
- A "propagation" can be seen along Cells 13 – 28

Key Takeaways

1. Requests only non-invasive measurements (U, I, B)
2. No prior knowledge needed
3. Successful reconstruction of resistivities from real measurement data
4. Converges within 20 steps
5. Permits a scalable mesh precision due to adjoint method (> 1000 variables)

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